PROCEEDINGS OF SCIENCE

The $B \to D^* l \nu$ form factor from lattice QCD

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We present a calculation of the zero-recoil form factor for $B \to D^* l \nu$ using a 2+1 improved staggered action for the light quarks (on the MILC configurations), and the Fermilab action for the heavy quarks. The Fermilab double-ratio method is used to compute various three point functions, and, guided by heavy quark effective theory, this allows us to construct the needed form factor.

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1. Introduction

The CKM element V_{cb} is important for the phenomenology of flavor physics in determining the apex of the unitarity triangle in the complex plane. For example, the Standard Model prediction of ε_K depends sensitively on V_{cb} (where it appears to the fourth power), and the present lattice errors on this quantity contribute errors to ε_K of the same size as those due to B_K, the kaon mixing parameter which has been the focus of much recent work [1, 2, 3]. It is possible to determine $|V_{cb}|$ from both inclusive and exclusive semileptonic B decays, and they are both limited by theoretical uncertainties. The inclusive method makes use of the heavy quark expansion [4, 5], but is limited by the breakdown of local quark-hadron duality, the errors of which are difficult to estimate. The exclusive method requires reducing the uncertainty of the form factor $\mathscr{F}_{B\to D^*}$, which has been calculated using lattice QCD in the quenched approximation [6]. Given the phenomenological importance of this quantity we have revisited this calculation of $\mathscr{F}_{B\to D^*}$ using the 2+1 flavor MILC lattices with improved light staggered quarks [9]. The quenching error is thus eliminated, and the systematic error associated with the chiral extrapolation is reduced significantly. The results given in these proceedings are preliminary, and no final numbers will be quoted. The point of this work is to describe the calculation, and to explain what remains to be done.

2. Obtaining $|V_{cb}|$

The differential rate for the semileptonic decay $\overline{B} \to D^* l \overline{\nu}_l$ is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2-1}\,\mathscr{G}(w) |V_{cb}|^2 |\mathscr{F}_{B\to D^*}(w)|^2 \tag{2.1}$$

where $w = v' \cdot v$ is the velocity transfer from the initial state to the final state, $\mathscr{G}(w)$ is a kinematic factor and $\mathscr{F}_{B\to D^*}$ is a matrix element which must be calculated nonperturbatively. This matrix element is a combination of several form factors, but at zero recoil it simplifies to $\mathscr{F}_{B\to D^*}(1) = h_{A_1}(1)$, which is sufficient to determine $|V_{cb}|$ from experiment. Heavy quark symmetry plays an important role in constraining $h_{A_1}(1)$, leading to the heavy quark expansion [10, 11]

$$h_{A_1}(1) = \eta_A \left[1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{2m_c 2m_b} - \frac{\ell_P}{(2m_b)^2} \right], \tag{2.2}$$

up to order $1/m_Q^2$ and where η_A is a factor which matches QCD and heavy quark effective theory (HQET). The ℓ's are long-distance matrix elements of the heavy quark effective theory. The above results were generalized to lattice gauge theory in [12].

Hashimoto et al [6] realized that these ℓ's could be computed precisely by making use of the double ratios of various matrix elements at zero recoil. For example, the double ratio $\mathscr{R}_1$ is

$$\mathscr{R}_1 = \frac{\langle D^* | \bar{c}\gamma_4 b | \overline{B}^* \rangle \langle \overline{B}^* | \bar{b}\gamma_4 c | D^* \rangle}{\langle D^* | \bar{c}\gamma_4 c | D^* \rangle \langle \overline{B}^* | \bar{b}\gamma_4 b | \overline{B}^* \rangle} = |h_1(1)|^2, \tag{2.3}$$

$$\begin{tikzcd}
\text{lattice} \arrow[dr, "\rho/\eta"] \arrow[dd, "\rho"'] & \\
& \text{HQET} \arrow[dl, "\eta"] \\
\text{QCD} &
\end{tikzcd}$$

Figure 1: Diagram illustrating how the matching factors ρ, η and ρ/η match lattice gauge theory and HQET to QCD, and to each other.

and yields the form factor,

$$h_1(1) \;=\; \eta_V \left[1 - \ell_V \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right],$$

and this gives the HQET parameter ℓ_V. Statistical fluctuations in the numerator and denominator are highly correlated and therefore largely cancel in the ratio. Also, the normalization uncertainty in the lattice currents largely cancels in the ratio. Thus, all uncertainties scale as $\mathscr{R} - 1$ rather than as $\mathscr{R}$.

The other two ℓ's can be determined from the following double ratios:

$$\mathscr{R}_+ = \frac{\langle D|\bar{c}\gamma_4 b|\overline{B}\rangle \langle \overline{B}|\bar{b}\gamma_4 c|D\rangle}{\langle D|\bar{c}\gamma_4 c|D\rangle \langle \overline{B}|\bar{b}\gamma_4 b|\overline{B}\rangle} = |h_+(1)|^2, \tag{2.4}$$

$$\mathscr{R}_{A_1} = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle \langle \overline{B}^*|\bar{b}\gamma_j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma_j\gamma_5 c|D\rangle \langle \overline{B}^*|\bar{b}\gamma_j\gamma_5 b|\overline{B}\rangle} = \left|\check{h}_{A_1}(1)\right|^2, \tag{2.5}$$

which give

$$h_+(1) \;=\; \eta_V \left[1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \tag{2.6}$$

$$\check{h}_{A_1}(1) \;=\; \check{\eta}_A \left[1 - \ell_A \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]. \tag{2.7}$$

Note that the lattice double ratios must be matched to the continuum using

$$\rho\sqrt{R_{lat}} = \sqrt{\mathscr{R}_{cont}} = h(1) \tag{2.8}$$

where the ρ factors can be calculated perturbatively. Given the ρ factors we can form the ratio ρ/η in order to match the lattice to the continuum HQET. The role of the various matching coefficients is illustrated in Figure 1.

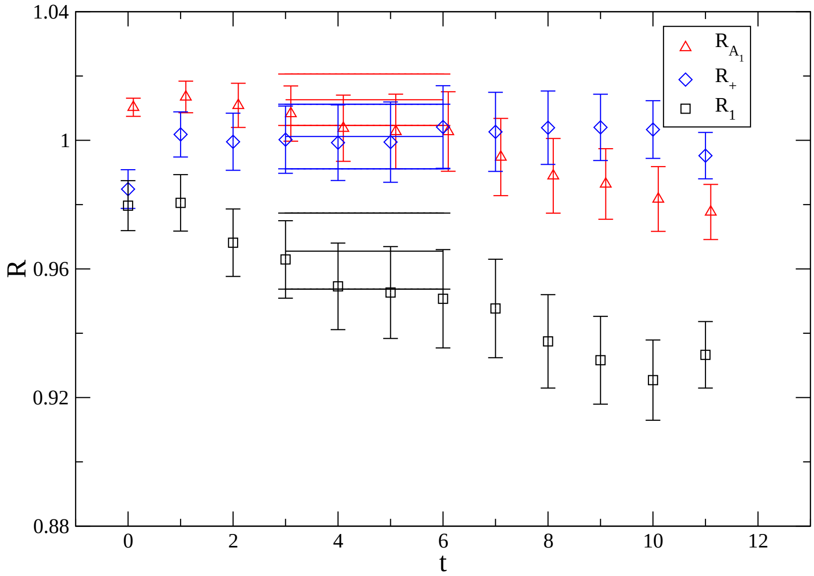

Figure 2: The three double ratios needed to determine $\mathscr{F}_{B\to D^*}$, as well as the correlated fits to their plateaus.

3. Lattice calculation

The lattice calculation was done on the MILC coarse lattices ($a \approx 0.125$ fm) where the light quarks were computed with the “asqtad" action. The heavy quarks were computed using the clover action with the Fermilab interpretation in terms of HQET [7]. We have looked at three light masses at the full QCD points, $m_{valence} = m_{sea}$. The lightest such mass was around $m_s/7$. The perturbative matching coefficients (the ρ factors) have been calculated by Nobes [8], but have not yet been incorporated into the analysis. In the following we will take the various ρ's and η's to be equal to one. The measured double ratios are shown in Figure 2, along with the fits to the identified plateau region.

In order to extract the ℓ's from the double ratios it is convenient to fit to the following quantity:

$$\frac{\rho\sqrt{R}}{\eta} = \frac{h}{\eta} = 1 - \frac{1}{4}\Delta^2(c^{(2)} + \frac{1}{2}c^{(3)}\Sigma), \tag{3.1}$$

where

$$\Delta = \frac{1}{am_{2c}} - \frac{1}{am_{2b}}, \qquad \Sigma = \frac{1}{am_{2c}} + \frac{1}{am_{2b}}. \tag{3.2}$$

and the coefficients $c^{(2)}$ and $c^{(3)}$ are HQET parameters at order $1/m_Q^2$ and $1/m_Q^3$, respectively. This fit allows the determination of three of the four coefficients that enter $\mathscr{F}_{B\to D^*}$ at order $1/m_Q^3$ in addition to the three coefficients of order $1/m_Q^2$. Fits using this formula are shown in Figure 3. It should be emphasized that these are highly correlated data points, and the fits are correlated fits

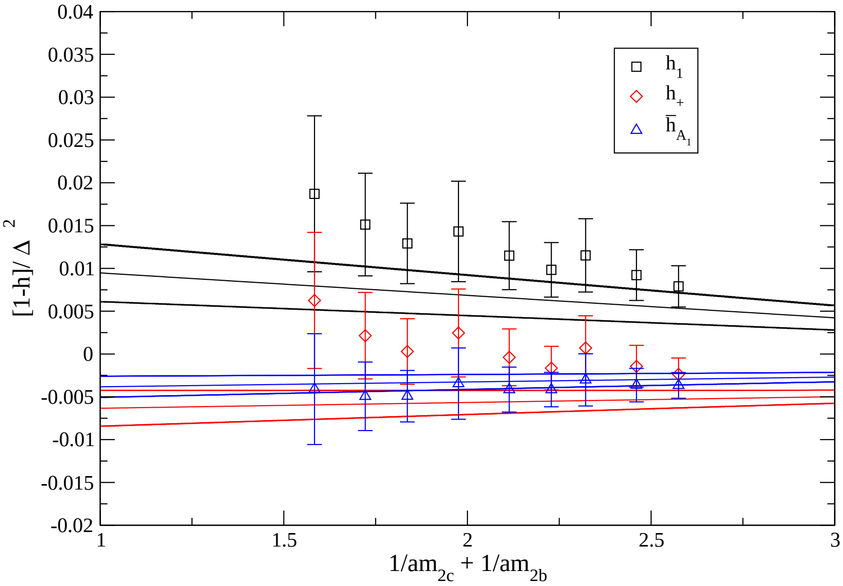

Figure 3: Heavy quark extrapolation for the three double ratios

with $\chi^2/\mathrm{dof} \approx 1$, even though they look poor by the standard of uncorrelated fits to uncorrelated data.

The chiral perturbation theory (ChPT) for heavy light mesons with a Wilson-like heavy quark and a staggered light quark was worked out by Aubin and Bernard in [13], and the ChPT relevant for $B \to D^*$ was done in [14]. The expression needed for the chiral fit is

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \;=\; & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\Bigg[\frac{1}{16}\sum_B \left(2\overline{F}_{\pi_B} + \overline{F}_{K_B}\right) - \frac{1}{2}\overline{F}_{\pi_I} + \frac{1}{6}\overline{F}_{\eta_I} \\
& + a^2\delta_V' \left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta_V'}^2)}\overline{F}_{\pi_V}\right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta_V'}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left. + \frac{m_{S_V}^2 - m_{\eta_V'}^2}{(m_{\eta_V}^2 - m_{\eta_V'}^2)(m_{\eta_V'}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V'}\right) + (V \to A)\Bigg],
\end{aligned}
\tag{3.3}
$$

where $\overline{F}_j \equiv F\left(m_j, -\Delta^{(c)}/m_j\right)$, and

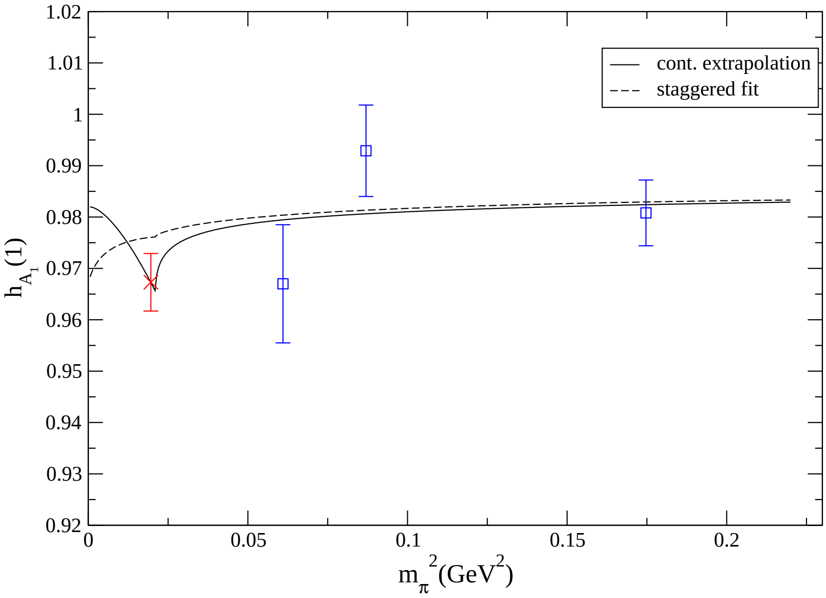

Figure 4: Chiral extrapolation, where the dotted line is the fit to staggered ChPT and the solid line is the continuum extrapolation. The “x" is the extrapolated physical light mass point.

$$\begin{aligned} F\,(m_j, x) \;=\; & \frac{m_j^2}{x}\left\{ x^3 \ln\frac{m_j^2}{\Lambda^2} - \frac{2}{3}x^3 - 4x + 2\pi \right. \\ & \left. -\sqrt{x^2-1}(x^2+2)\left(\ln\left[1-2x(x-\sqrt{x^2-1})\right] - i\pi\right)\right\} \\ & \longrightarrow (\Delta^{(c)})^2 \ln\left(\frac{m_j^2}{\Lambda^2}\right) + \mathscr{O}[(\Delta^{(c)})^3], \end{aligned} \tag{3.4}$$

with g_π the D^*-D-π coupling, $\Delta^{(c)} = m_{D^*} - m_D = 142$ MeV and X_A is a term that is independent of the light quark masses. Other than the staggered taste-splittings and hairpin coefficients that have been obtained separately from fits to MILC data in the pseudoscalar sector [9], this formula has only one unknown. Performing the fit to the above formula gives the dotted line in Figure 4. The solid line is the continuum extrapolated result obtained by taking a^2 to zero in this expression. Note that the physical pion mass occurs near a “cusp" resulting from threshold effects, where an intermediate D and π can go on-shell (since $\Delta^{(c)} = m_{D^*} - m_D = 142$ MeV, is slightly greater than the pion mass). This cusp and the effects of “staggering" it are discussed in more detail in [14].

4. For the future

The plan for the future is to add statistics by running on multiple time sources, and to run on additional lattice spacings generated by MILC and the Fermilab lattice group in order to determine the lattice spacing dependence. We will incorporate the matching coefficients into the analysis, and of course, we will do a careful analysis of all the systematic errors in order to report a final number for the $B \to D^* \ell \nu$ form factor.

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